Government Deficits, *ex post* Real
Long-term Interest Rates and Causality*

Richard J. Cebula

1. Introduction


The budget deficit-interest rate relationship is usually couched within either the IS/LM framework or a loanable funds framework. Within the IS/LM framework, the traditional view is that the IS curve is negatively sloped and the LM curve is positively sloped. In any case, given IS/LM stability, a deficit-financed increase in real Government purchases of goods and services shifts the IS curve upwards, generating — among other things — higher interest rates. This paradigm clearly implies a direction of causality from increased deficits to interest rates. Similarly, the loanable funds model also implies that causality flows from the Government budget deficit to

* Georgia Institute of Technology, Atlanta (USA).

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the long-term interest rate (Barth, Iden and Russek 1984, Hoelscher 1986, Cebula 1988).

However, it is argued in the present study that the direction of causality between real long-term interest rates and the budget deficit may well be the reverse, i.e., that real long-term interest rates may cause the budget deficit. This reverse causality seems plausible for at least two reasons. First, much of the federal Government’s debt is financed or re-financed in just a few short years. For example, the average maturity of the federal Government debt ranges from a low of 2.58 years to a high of 6.1 years during the period from 1973 to 1996. This means that within a period of just two or three years a majority of the national debt may be re-financed. Thus, a rise in the real interest rate quickly translates into an increase in the real interest payments that must be made during any given period to service the national debt. In point of fact, net interest on the national debt in 1996 represented approximately 15.0% of the total budget of the federal Government in the United States.  

Second, according to conventional macroeconomic theory, a rise in the real interest rate should lead to a fall in real economic growth as aggregate investment and durable consumption demand fall. Such a decline in economic growth might well lead to higher unemployment and thus to an increase in the Government’s cyclical deficit through reduced tax collections and increased transfer payments (such as unemployment benefits); furthermore, to the extent that policy-makers are sensitive/responsive to higher unemployment, the structural deficit may also be increased in an effort to combat that higher unemployment.

The purpose of this study is to analyze empirically the direction of causality between the federal Government budget deficit and the real long-term interest rate in the United States. First, we study the direction of causality between the total federal Government budget deficit and the real long-term interest rate. Then, we dichotomize the total federal budget deficit into its structural and cyclical deficit components and, using a multivariate causality test, determine the direction of causality among the real long-term interest rate, the unemployment rate and the structural deficit.

1 On the relationships among interest rates, the national debt and budget deficits as a proportion of GNP, see Domar (1947) and Pasinetti (1989).

2. The empirical analysis

The initial step in the analysis is to define the basic variables under investigation. Consistent with most of the existing studies, the initial measure of the total budget deficit is TDY, defined here as the ratio of the seasonally adjusted total National Income and Product Accounts federal Government budget deficit in quarter t to the seasonally adjusted GDP in quarter t, expressed as a percent. The real long-term interest rate yield in quarter t, R20, is defined as the nominal average interest rate yield on 20-year US Treasury bonds during the quarter (expressed as a percent per annum) minus the actual inflation rate of the consumer price index in the quarter (expressed as a percent per annum). Thus, R20, is an ex post real interest rate yield. This formulation for R20 as the ex post real interest rate is consistent with the analysis in Cukierman and Melzer (1989) and with the empirical studies by Evans (1985), Belton (1992 and 1993) and Cebula (1991a and 1991b). Moreover, attempting to formulate an ex ante real interest rate can potentially be quite problematic since an appropriate measure of inflationary expectations may be difficult to find (Swamy, Kolluri and Singamsetti 1990). For example, use of survey data such as the Livingston data is unsatisfactory because “the heuristics people have available for forming expectations cannot be expected to automatically produce expectations that come anywhere close to satisfying the normative constraints on subjective probability judgements provided by the Bayesian theory” (Swamy, Kolluri and Singamsetti 1990, p. 1013). On the other hand, as they observe (ibidem) it may be reasonable and useful to use “a distributed lag on actual price inflation” in order to attempt to generate an expected inflation proxy. In any case, focusing on the ex post real interest rate not only is consistent with many previous studies but also permits us to avoid the potential problems associated with finding a satisfactory expected inflation measure.

The next step in the empirical investigation is to determine the appropriate form of the causality test. In order to avoid spurious regression results, we first test for stationarity of the variables using the Augmented Dickey-Fuller (ADF) test, adopting the Schwarz-Bayesian criterion to determine the optimal lag length of the autoregressive term. The data are quarterly; the time period for this...
study is 1973.2-1996.3. We begin with 1973.2 since this quarter marks the collapse of the system of fixed exchange rates (Bretton Woods). We end with 1996.3 in order to make the study as current as available data now permit. The ADF test reveals that both \( TDY \) and \( R20 \) are non-stationary in levels. Thus, \( DTDY \), the first difference of \( TDY \), and \( DR20 \), the first difference of \( R20 \), are used in the empirical analysis. The ADF statistics are \(-10.20 \) and \(-7.91 \) for \( DTDY \) and \( DR20 \) respectively, which implies that both of the variables (the deficit and real interest rate) are stationary in first differences. The results of the Phillips-Perron test are entirely consistent with these findings.  

The next step is to test for cointegration between \( TDY \) and \( R20 \). We adopt the cointegration test developed by Johansen and Juselius (1990) and find that the cointegration vector \((-1.08,-2.99)\) in equation (1) is rank 1, indicating that the variables are cointegrated:  

\[
z_t = R20_t - 2.99TDY_t \tag{1}
\]

This particular vector maximizes the probability of stationarity in the system. The likelihood ratio test statistic is 43.98, which permits rejection of the null hypothesis of no cointegration at the 99% confidence level. The cointegrating vector, \( z \), indicates that there is a long-term, positive relationship between the real long-term interest rate and the total federal Government budget deficit.

With the variables being integrated as \( I(1) \) but cointegrated as \( CI(0) \), the error-correction model is used to test for the direction of causality. Thus, we test for causality by estimating the parameters of equations (2) and (3):

\[
DR20_t = a_0 + \sum_{i=1}^r a_i DR20_{t-i} + \sum_{j=1}^s a_j \text{DTDY}_{t-j} + a_2 z_{t-1} + u_t \tag{2}
\]

\[
\text{DTDY}_t = b_0 + \sum_{i=1}^r b_i DR20_{t-i} + \sum_{j=1}^s b_j \text{DTDY}_{t-j} + b_2 z_{t-1} + v_t \tag{3}
\]

where \( DR20, DTDY \), and \( z \) are as described above and \( a_i \) and \( b_i \) are stochastic error terms. The lag-lengths \( r \), \( s \), \( r \), and \( s \) are determined using the Schwarz-Bayesian criterion (SBC).

The budget deficit is said to 'cause' the real long-term interest rate if the sum of the \( a_i \)'s is significant or if \( a_2 \) is statistically significant and the sum of the \( b_i \)'s are not significant and \( b_2 \) is not significant. Similarly, the real long-term interest rate 'causes' the budget deficit if the sum of the \( b_i \)'s is significant or if \( b_2 \) is significant and the \( a_i \)'s are not significant and \( a_2 \) is not significant. Finally, there is 'bi-directional causality' if the \( a_i \)'s or \( a_2 \) is significant and if \( b_i \)'s or \( b_2 \) is significant.

The causality test was performed using OLS. As observed earlier, the lag-length was determined using the SBC, and the residuals were tested against the hypothesis of serial correlation using both the Lagrange-multiplier test and the Box-Pierce Q-statistic. These statistic tests are not reported here but indicate the absence of any significant serial correlation.

The estimated parameters for equations (2) and (3) are:

\[
DR20_t = -0.55 + 0.35 DR20_{t-1} - 0.27 DR20_{t-2} + 0.36 DR20_{t-3} \quad (-2.98) \quad (-2.13) \quad (+3.22) \tag{4}
\]

\[
-0.28 DTDY_{t-1} - 0.11 z_{t-1} \quad (-1.08) \quad (-1.59)
\]

\[
\text{adj. } R^2 = 0.19
\]

\[
\text{DTDY}_t = 0.68 + 0.083 DR20_{t-1} + 0.06 DTDY_{t-1} + 0.25 DTDY_{t-2} \quad (+2.83)
\]

\[
-0.39 z_{t-1} \quad (+1.67) \quad (+0.60)
\]

\[
\text{adj. } R^2 = 0.34
\]

where the terms in parentheses beneath the coefficients are \( t \)-values. \( z \) is the cointegration vector, written so that the coefficient of the dependent variable is one.  

In equation (4), neither the coefficient on \( z \) nor the coefficient on \( DTDY_{t-1} \) is statistically significant at even the 10% level, im-

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2 In general, the Phillips-Perron test confirms all of the ADF results obtained in this study. The Phillips-Perron results will be supplied upon written request.

3 These results will be supplied upon written request.

4 For example, in equation (4), \( z = R20, -2.99TDY \), and in equation (5), \( z = TDY, -0.39R20 \).
plying that (in contrast to the predictions of the IS/LM and loanable funds models) budget deficits do not have a long-run causal impact on the real long-term interest rate.

The coefficient on \( z_{\Delta} \) in equation (5) is negative and statistically significant at the 1% level, implying that the real long-term interest rate does, in the long-run, cause the budget deficit. This significant negative sign indicates a positive causal effect such that a rise in the \textit{ex post} real long-term interest rate causes a rise in the total federal Government budget deficit during the 1973.2-1996.3 period. Thus, it appears that the positive correlation between the \textit{ex post} real long-term interest rate and the total budget deficit found in previous, earlier studies of the United States may be a result of the effect of the real long-term interest rate on budget deficits, not \textit{vice versa}. This result might well arise for either or both of the reasons summarized in the Introduction in this study.

To gain further insight from this analysis, the total budget deficit measure is now dichotomized into its structural and cyclical components. Therefore, we can write:

\[
\text{DTDY}_t = \text{DSDY}_t + \text{DCDY}_t
\]

(6)

where \( \text{DSDY}_t \) is the first difference of \( SDY_t \), the ratio of the seasonally adjusted structural budget deficit in quarter \( t \) to the seasonally adjusted GDP in quarter \( t \) (expressed as a percent); and \( \text{DCDY}_t \) is the first difference of \( CDY_t \), the ratio of the seasonally adjusted cyclical budget deficit in quarter \( t \) to the seasonally adjusted GDP in quarter \( t \) (expressed as a percent). Furthermore, we observe that \( DCDY_t \) has in earlier studies been found consistently to be principally a function of \( DU_t \), the first difference of \( U_t \), the seasonally adjusted unemployment rate of the civilian labor force (expressed as a percent) in quarter \( t \) (Belton 1992 and 1993, Cebula 1988, 1991a and 1991b, Ostrosky 1990). Thus, we can write the following:

\[
\text{DTDY}_t = \text{DSDY}_t + \text{DCDY}_t
\]

(7)

Like \( TDY_t \) and \( R20_t \), \( SDY_t \) and \( U_t \) have unit roots (are not stationary in levels). Thus, we must use \( \text{DSDY}_t \) and \( \text{DU}_t \) in our model rather than the structural deficit and unemployment rate in levels; this is because using the Schwarz-Bayesian criterion, the respective ADF statistics for \( \text{DSDY}_t \) and \( \text{DU}_t \) are -11.34 and -6.75, both of which are significant at the 1% level.

With \( z = R20 - 2.9U - 2.85SDY \), the cointegration vector (1 \(-2.9 -2.85\)) is rank 1, indicating that the variables \( U, SDY, \) and \( R20 \), are cointegrated. This particular vector (1 \(-2.9 -2.85\)) maximizes the probability of stationarity in this system. To supplement the above results and attest further to the robustness of the results, the likelihood ratio test statistic is 49.95. Thus, we can reject the null hypothesis of no cointegration at the 99% confidence level. Again, we express the error-correction term, the cointegration vector, such that the dependent variable has the coefficient of 1.5 Thus, we refer to equations (8), (9) and (10), which represent the multivariate error-correction model:

\[
\text{DR20}_t = a_0 + \sum_{i=1}^{I} a_i \text{DR20}_{t-i} + \sum_{j=1}^{J} a_j \text{DSDY}_{t-j} + \sum_{k=1}^{K} a_k \text{DU}_{t-k} + a_z z_{t-1} + u_t \quad (8)
\]

\[
\text{DSDY}_t = b_0 + \sum_{i=1}^{I'} b_i \text{DR20}_{t-i} + \sum_{j=1}^{J'} b_j \text{DSDY}_{t-j} + \sum_{k=1}^{K'} b_k \text{DU}_{t-k} + b_z z_{t-1} + v_t \quad (9)
\]

\[
\text{DU}_t = c_0 + \sum_{i=1}^{I''} c_i \text{DR20}_{t-i} + \sum_{j=1}^{J''} c_j \text{DSDY}_{t-j} + \sum_{k=1}^{K''} c_k \text{DU}_{t-k} + c_z z_{t-1} + w_t \quad (10)
\]

where \( u_t, v_t \), and \( w_t \) are the stochastic error terms in this model, and \( I, I', I'', m, m', m'', n, n', n'' \) are the lag-lengths, as determined by the SBC.

Furthermore, using \( \text{DU}_t \) in place of \( \text{DCDY}_t \) offers a major theoretical advantage. There is no theoretical reason to believe that the cyclical deficit should have any different effect on the real long-term interest rate than the structural or total deficit. Bond buyers cannot distinguish bonds sold by the Treasury to finance the structural deficit from those bonds sold by the Treasury to finance the cyclical deficit. If \( DCDY_t \) is used as a variable in the model, its coefficient should be the same as that on \( DSDY_t \), plus the effect of \( DU_t \), on the dependent variable. Since we expect \( DU_t \) to have an impact on \( \text{DR20}_t \), or \textit{vice versa}, it is important to include \( DU_t \) in the model. It is possible, if there is no relationship between \( TDY_t \) and \( R20_t \), that \( DTDY_t \) merely stands as a proxy for \( DU_t \) in (4) and (5) since changes in \( DU_t \) cause changes in \( DTDY_t \) and may be correlated with \( DR20_t \).
A priori, the cointegration vector seemingly suggests one possible direction of causality. The vector indicates that $R_{20}$ and $U_1$ are positively related. This is not consistent with the theory that $U_1$ causes $R_{20}$, because we would expect that an increase in $U_1$ would decrease the demand for loanable funds and lower real long-term interest rates. However, if the direction of causality is from the real long-term interest rate to unemployment, this cointegration vector is consistent with conventional macroeconomic theory: that is, we would expect a rise in the real long-term interest rate to cause a rise in $U_1$.

Estimating the parameters of our model using OLS (both the Lagrange-multiplier test and the Box-Pierce Q-statistic reveal no serial correlation) yields:

$$
DR_{20} = -1.40 + 0.35DR_{20,-1} - 0.41DR_{20,-2} + 0.37DR_{20,-3} - 0.20DR_{20,-4} + 0.30DSDY_{-1} + 0.48DU_{-1} - 0.060z_{-1} + 0.56DU_{-1} + 0.055z_{-1} - 0.52DU_{-1} + 0.056z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1}$$

$$
(2.88) \quad (3.24) \quad (3.21) \quad (1.76) \quad (-1.05) \quad (1.10) \quad (-1.09) \quad (2.57) \quad (-3.33) \quad (-2.16) \quad (-2.67) \quad (+3.77) \quad (+1.06)
$$

**adj. $R^2 = 0.18$**

$$
DSDY_t = -0.52 + 0.11DR_{20,-1} - 0.41DSDY_{-1} - 0.25DSDY_{-2} - 0.27DSDY_{-3} + 0.56DU_{-1} + 0.055z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.100DU_{-1} + 0.063z_{-1}$$

$$
(2.37) \quad (-3.33) \quad (-2.16) \quad (-2.67) \quad (+3.77) \quad (+1.06)
$$

**adj. $R^2 = 0.28$**

$$
DU_t = 1.02 + 0.07DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1} + 0.007DR_{20,-1} + 0.02DSDY_{-1} + 0.60DU_{-1} - 0.11z_{-1} + 0.100DU_{-1} + 0.063z_{-1}$$

$$
(3.95) \quad (0.39) \quad (8.10) \quad (-4.45)
$$

**adj. $R^2 = 0.65$**

For equation (11), the coefficient on $DSDY_{-1}$ is not statistically significant at even the 10% level. Coupled with the non-significance of the coefficient on $z_{-1}$, this finding allows us to reject the hypothesis that the structural budget deficit causes the ex post real long-term interest rate. This conclusion is entirely consistent with our previous results. In addition, the coefficient on $DU_{-1}$ is not statistically significant at even the 10% level, which allows us to reject the hypothesis that unemployment causes the real long-term interest rate.

The coefficient on $DR_{20,-1}$ in equation (12) is statistically significant at the 2% level, which allows us to accept the hypothesis that the ex post real long-term interest causes the structural deficit. The coefficient on $DU_{-1}$ is positive and statistically significant at the 1% level, allowing us to accept the hypothesis that the unemployment rate causes the structural deficit as well as the cyclical deficit.

Equation (13) reveals both a long-run and short-run causality such that a higher real long-term interest rate causes a rise in the unemployment rate. The support for this hypothesis is statistically significant at the 1% level for both the long and the short-run.

### 3. Conclusions

The preliminary results reported in equations (4), (5), (11), (12) and (13) strongly suggest that in the United States, over the 1973.2-1996.3 period, a rise in the ex post real long-term interest rate caused a rise in the federal government budget deficit, not vice versa, as found in a number of previously published earlier-period studies. This causality appears to arise from both the effect of the real long-term interest rate on the cost of financing (re-financing) the national debt and the effect the real long-term interest rate has on the unemployment rate. This is seemingly confirmed by both the causality running from $DR_{20}$ to $DSDY$, in (5) and the causality running from $DR_{20}$ to $DSDY$, in (12), in which we account for the effect of changes in the unemployment rate on $DSDY$, as well. In addition, we find the $DR_{20}$ causes $DU$, i.e., increases in the real long-term interest rate cause a rise in unemployment. There also may exist feedback onto the government budget deficit, as indicated by the causality running from $DU$ to $DSDY$, shown in equation (12). Thus, apparently, when a rise in the long-term interest rate causes a rise in unemployment, not only may this increase the cyclical deficit, but policy-makers also seem to respond by increasing the structural deficit as well in order to stimulate the economy and thereby combat unemployment.

It is stressed that the findings provided in this study are only preliminary in nature. What they may reveal nevertheless is that the relationship between government budget deficits and interest rates may be far more complex than previously believed.
Naturally, the policy implications of these results are potentially very significant. First, these results would not support the notion of crowding out, since there is no empirical evidence that a rise in the budget deficit causes a rise in the real long-term interest rate. Second, the result that a rise in the real long-term interest rate might cause a rise in the budget deficit has potentially major implications for the conduct of monetary policy. These possibilities notwithstanding, further work into the issue at hand is necessary.

APPENDIX

THE UNDERLYING FRAMEWORK

To demonstrate the underlying framework for the empirical analysis, consider the following intertemporal Government budget constraint:

\[ ND_{t+1} = ND_t + G_t + F_t + RND_t - T_t \]  

(A.1)

where:

- \( ND_{t+1} \) is the national debt in period \( t+1 \)
- \( ND_t \) is the national debt in period \( t \)
- \( G_t \) = Government purchases in period \( t \)
- \( F_t \) = Government transfer payments in period \( t \)
- \( R_t \) = Average interest rate on the national debt in period \( t \)
- \( T_t \) = Government tax and other revenues in period \( t \)

The total budget deficit for period \( t \) (\( TD_t \)) is the difference between \( ND_{t+1} \) and \( ND_t \):

\[ TD_t = ND_{t+1} - ND_t = G_t + F_t + RND_t - T_t \]  

(A.2)

The total amount of interest paid on the national debt is a function of the current interest rate, the past interest rate, the size of the national debt and the proportion of the national debt that is financed or re-financed at the current interest rate. For simplicity, we may write:

\[ RND_t = CR_t ND_t + R_{t-1} (1 - a) ND_t \]  

(A.3)

where \( CR_t \) is the average interest rate on current (i.e., period \( t \)) Government borrowing, \( R_{t-1} \) is the average interest rate on the national debt in period \( t-1 \) and \( a \) is the proportion of the national debt that is either newly financed (i.e., a current budget deficit) or re-financed old debt. From equation (A.3), it follows that the total interest paid on the Government debt, ceteris paribus, rises when \( CR_t > R_{t-1} \), which in turn implies that \( R_t > R_{t-1} \). In this context (in contrast to the traditional IS/LM or loanable funds models), causality flows from interest rates to deficits.

Furthermore, one can incorporate the effect of \( U_t \), the unemployment rate in period \( t \), into the model, as follows:

\[ F_t = F_t (U_t), F'_t (U_t) > 0 \]  

(A.4)

\[ T_t = T_t (U_t), T'_t (U_t) < 0 \]  

(A.5)

A rise in \( U_t \) is expected to raise \( F_t \) and to decrease \( T_t \), thereby raising \( TD_t \). Thus, the total deficit is a function not only of \( R_t \) but also of \( U_t \).

If we disaggregate the total deficit into its structural (\( SD_t \)) and cyclical (\( CD_t \)) components, it is expected that a higher value for \( R_t \) may raise both \( SD_t \) (by increasing anticipated debt service) and \( CD_t \) (by automatically increasing unanticipated debt service and, to the extent that higher real interest rates reduce the pace of economic activity, by automatically lowering tax revenues and increasing transfer payments).

Furthermore, (A.4) and (A.5) imply that the effect of \( U_t \) on \( TD_t \) may be expressed in terms of an impact on \( CD_t \); however, to the degree that fiscal policy-makers are responsive to \( U_t \), \( SD_t \) is also likely to be an increasing function of \( U_t \) through the exercise of discretionary fiscal policy.

REFERENCES


