Endogenous Growth, Economic Fluctuations and Capital Accumulation*

MAURO GALLEGATI

1. Introduction

In the Keynesian tradition, models of cyclical fluctuations normally analyze the dynamics of the cycle around an exogenous trend. The characteristic feature of such analysis is the splitting of theoretical factors and their related time series into two separate components: short-term variables, which are endogenously determined in order to explain cycles, and long-term variables, which explain trend but are exogenously determined. This separation is based on the assumptions that changing flows of new investment are important in the determination of current income but play a minor role in capital stock, and that the output-capital ratio or the labour-capital ratio are constant. In the Keynesian approach these assumptions have provided the basis for accelerator-multiplier models which are able to explain either cyclical fluctuations or some simplified growth path, but not both (Passinetti, 1981). Thus, the combining of cyclical fluctuations and growth in the same model is still one of the main problems to be solved in the reappraisal of Keynesian results.

In recent decades, the economic performance of the industrialized countries has offered various reasons for dissatisfaction with the Keynesian tradition. In particular, the Keynesian faith in high rates of growth and the consequent shift to the short-term control of

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* Università "G. D'Annunzio", Dipartimento di metodi quantitativi e teoria economica, Pescara (Italy).

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demand fluctuations proved to be ill-timed. Secondly, the main statistical sources demonstrate that capital-labour ratios are not constant and that the impact of investment on capital stock is not negligible.

The growth models that dominated mainstream economics during the 1960s are not very helpful in this regard. Solow's models analyzed the growth path of aggregate economic systems but not their related cyclical fluctuations. Nor has the gap been filled by the new theoretical literature on cycles and growth. In fact, while the Lucas (1988) and Romer (1986, 1990) approaches focus on long-term growth, and thus obscure short-term fluctuations, the Real Business Cycle approach (Kydland and Prescott, 1982) collapses growth processes into "random walk" thus obscuring long-term fluctuations.

This paper aims to remedy the shortcomings of the above-mentioned models by examining imperfect information and financial constraints in a New Keynesian setting (Maniw and Romer, 1990: Symposium on the Journal of Economic Perspectives, 1993). Amongst other things, capital-market imperfections and related information asymmetries induce agents to be risk averse, and give rise to changes in the net worth of firms and banks. It is asymmetric information that binds together banks' lending and firms' borrowing; hence they interact effects on the aggregate amount of production and investment, and may cause economic instability. Our theory is that these effects on financial constraints are large and persistent enough to constitute an endogenous determination of cyclical fluctuations as well as of trend.

2. A New Keynesian Macroeconomic Model

This section illustrates a New Keynesian macroeconomic model, which develops a previous model by Greenwald and Stiglitz (1993a). I shall analyze the process of fluctuating growth by introducing technical progress and capital accumulation and by modifying the Greenwald-Stiglitz framework in order to take account of the demand for investment and capital accumulation. This new model allows for a complete set of dynamic behaviour. In fact, the model's reduced form is non-linear: a necessary condition in order to obtain stochastic fluctuations or self-sustained cycles or a combination of the two.

2.1 Firm Behaviour and Investment

In this model there are n perfectly competitive firms whose output decisions are made by managers who are averse to the possibility of bankruptcy. Because of information imperfections, firms' ability to raise equity funds in external capital markets is restricted and, since futures markets do not exist and capital inputs (i.e. investment) are paid before output is sold, any decision to produce is a risky investment decision. Firms face a real wage \( w \) and at that wage they can hire as much labour as they want up to a full employment position (for the sake of simplicity I assume \( w \) is the reservation wage and wages are paid at the end of the production period). Firms can also borrow as much as they want in the credit market at terms which must yield the lender an expected real return of \( r \) (in equilibrium, this represents the households' intertemporal preference) while equity rationing holds true.

In each period the total equity base evolves according to:

\[
    s_t = y_t + \left( \frac{P_{t+1}^F}{P_t} \right) (s_{t-1} - w_t - d_t) + \mathcal{R}_t
\]

where: \( a \) = the real equity position of firms; \( y \) = the real level of output; \( p \) = the price level, where the suffix \( t \) stands for expected; \( r \) = the real rate of interest; \( w \) = the real wage; \( l \) = employment; \( d \) = the
amount of bank loans borrowed by firms at the beginning of the production process. In particular, I assume a rational expectations setting which implies that \( p_0 = p_1 | \Omega_{x,t} \). Total output is equal to:

\[
y_t = qk_t.
\]

Dividing (2) by \( l \), labour productivity, \( \pi = y/l \), equals the output-capital ratio, \( q \), times the capital-labor ratio, \( \psi = k/l \):

\[
\pi = \psi q
\]

or

\[
y_t = l_1 \psi q_t.
\]

Investment demand, \( i_t \), is the minimum between a multiple of the equity level of the previous period, and a full employment ceiling, \( \bar{r}^* \):

\[
i_t = \min(ba_{t-1}, \bar{r}^*)
\]

where \( b > 1 \). Capital stock evolves according to:

\[
k_t = k_{t-1}(1 - \delta) + i_t,
\]

where \( \delta \) is the physical capital depreciation rate.

It is a well-established finding of the New Keynesian asymmetric information literature (Greenwald and Stiglitz, 1993b) that, in each period, output is determined by the level of equity, and that movements of output over time will be driven by movements in the level of equity. This result is a consequence of the hypothesis of asymmetric information: a hypothesis under which, in fact, the Modigliani-Miller theorem no longer holds true, and a financing hierarchy can be envisaged which ranges from low-cost finance (internal funds) to high-cost funds (external finance). Accordingly, the aggregate supply function is no longer a function of the price level alone, as the traditional formulation would have it, but of the equity base as well. Furthermore, I assume that demand shocks affect the expected price level, thus generating short-term fluctuations.

### 2.2 The Loan Market

Firms finance their investment activity partly from internal funds and partly from external funds borrowed from banks. Their investment cost may be not completely covered by their equity base. Thus, if the internal funds' availability of the firm is lower than its demand for investment, it must seek external finance by borrowing from the competitive banking system. Now the existence of asymmetric information on the credit market limits the firm's access to external funds (Stiglitz and Weiss, 1981). With asymmetric information, the effective interest rate charged by banks is \( r_t = R_t + \Theta_t \), where \( \Theta_t \) is the lemon premium (Akerlof, 1970) and \( R_t \) is the rate of interest risk-free. Thereafter I assume, without any loss of generality since wages are paid at the end of the production period, that the firms' demand for external funds, \( d_t \), is limited to:

\[
d_t = l_1 - a_{t-1}
\]

while the supply of loans, \( f_t \), is equal to the interest rate times a credit multiplier, \( l_{t-1} \):

\[
f_t = l_{t-1} r_t
\]

Note that the lemon premium can be endogenized as a function of the equity base. In this case, it varies with the business cycle, and is higher in depression than in expansion. A situation of self-fulfilling prophecy can be envisaged if we link the return on a loan to expectations. If banks expect a bad period, their willingness to lend will decrease, or else they will charge a higher lemon premium. Evidently, if risk increases the expected loss will increase, thus shifting the supply curve downwards, which means a lower amount of credit, and therefore less investment and, possibly, a higher interest rate. Below, I argue that the supply of finance is linear without the possibility of credit rationing being taken into account.

The equilibrium condition requires that:

\[
d_t = f_t.
\]

Substituting (7) and (8) in (9) yields

\[
r_t = \frac{1}{1 - a_{t-1}}(b - 1)
\]
which implies that the rate of interest will rise whenever \( b \) or the lagged equity base increase, and the credit multiplier decreases.

2.3 The Macroeconomic Equilibrium

Substituting (2), (4)-(5) and (10) in (1), assuming realized expectations (i.e. a deterministic setting which implies that \( p_{t}^{*} = p_{t} \)), and (5) in (6), yields the system:

\[(1a) \quad a_{t} = \varphi_{2} a_{t-1} + \varphi_{1} a_{t-1} + \varphi_{0} = \Gamma \]

\[(6a) \quad k_{t} = (1 - \delta)k_{t-1} + b a_{t-1} \]

where \( \varphi_{0} - k_{t-1}([1-\delta(q-\frac{w}{\psi})]) > 0, \varphi_{1} + h_b(q-\frac{w}{\psi}) > 0, \varphi_{2} - q - \frac{1}{\delta} > 0.1 \)

The first-order non-linear difference equation (1a), which is mapped in Figure 1 in the \( a_{t-1} - a_{t} \) axes, is a parabola with a downwards concavity. There are two steady state values:

\[(11) \quad a^{*} = \frac{1}{2 \varphi_{2}} \left[ - (\varphi_{1} - 1) \pm \sqrt{(1 - \varphi_{1})^{2} - 4 \varphi_{0} \varphi_{2}} \right] \]

but only one of them is economically meaningful, i.e. \( a^{*} \).

Differentiating \( \Gamma \) with respect to \( a \), we obtain

\[(12) \quad \Gamma(a^{*}) = 1 - \sqrt{(1 - \varphi_{1})^{2} - 4 \varphi_{0} \varphi_{2}} \leq 0, \]

if \( \varphi_{1} - 2 + \varphi_{2} \geq 4 \varphi_{0} \varphi_{2} \). Of course, if \( \Gamma > 0 \), convergence to the steady state is monotone, while if \( \Gamma < 0 \) convergence to the steady state follows an oscillating path; a deterministic cycle of multiple periodicity occurs if the slope of the curve \( \Gamma \) is sufficiently negative when it crosses the 45° line (Grandmont, 1985; Goodwin, 1990).

\[1 \text{ For the sake of simplicity (without any loss of generality as to the dynamic behaviour involved) we will analyze equation (11) alone, assuming } \Delta k = 0. \text{ An analysis of the non-linear map (1a)-(6a) is in Gardini (1993).} \]

If \( \Gamma > 0 \), the Slutsky-Frisch approach (note the different paths followed by the system toward equilibrium originated from the work of Frisch, 1933, and Slutsky, 1937, respectively) can be usefully applied if we take into account the fact that

\[ p_{t}^{*} = p_{t} + \varepsilon_{t} \]

i.e. the expected price is affected by a stochastic disturbance \( \varepsilon_{t} \), with zero mean and variance \( \sigma^{2} \). Vice versa, when a deterministic path may emerge, due to chaotic dynamics, cycles follow an erratic path, i.e. their path is stochastic even if it is produced by a deterministic system.

In the deterministic setting of the model, a period of increasing activity leads to higher profits, higher demand for loans and therefore a higher interest rate. This last effect lowers the equity level by increasing debt commitments and reduces activity, therefore, since the equity base is lower, the demand for loans and the interest rate will decrease, thereby stimulating the equity level, production and profits. The cycle can therefore start again.

Note that the \( \Gamma \) curve is parametrized to \( k_{t-1} \). This means that, when capital accumulates, the curve will shift upwards increasing the level of \( a^{*} \). Without entering into technical details, consider Figure 2, in which I represent the evolution of the equity base over time. The figure shows that the fluctuations accompanying adjustment toward
the equilibrium position are coupled with growth because capital accumulation shifts the $\Gamma$ curve from $k$ to $k^*$ without affecting either the vertex or the slope of the $\Gamma$ curve if the rate of interest is kept constant (a quite intuitive result, since capital accumulation simply reproduces the dynamic behaviour of the system on a larger scale). As will become clearer below, this recalls Schumpeter's dichotomy between growth and development, the former being a quantitative increase in the system, while the latter requires structural change, i.e. a qualitative change in the working of the system because of technical change.

In any case, the growth process due to capital accumulation tends to zero because of two effects: $i$) if the system approaches the steady-state equilibrium position (i.e. when there are no endogenous fluctuations), capital accumulation tends to zero since equity-base change tends to zero as time goes to infinity; $ii$) there is a full employment barrier ($k^*$ in Figure 2), which can be overcome only if the capital-labour ratio increases. In the next section, I analyze the dynamic properties of the model when capital accumulation and technical change are set to zero, while Section 4 conducts to longer run analysis.

3. Fluctuations and Cycles

We have already seen that, depending on the shape of the curve $\Gamma$, the model can give rise to fluctuations, i.e. changes in output due to shocks, and deterministic cycles, i.e. self sustained movements around a repulsive point within a stable domain. Elsewhere (Gallegati, 1993), I have demonstrated that the system may assume different dynamic behaviour depending on the stochastic evolution of the parameter values; values which, ultimately, affect the shape of the curve $\Gamma$. Below, I will analyze both types of behaviour.

3.1 Responses to Shocks and Fluctuations

In Section 2.1 we saw that the equity level can be affected by stochastic disturbances in expected price. Let us assume $\Gamma \geq 0$, without $\Gamma^* \ll 0$. If an adverse shock affects the expected price, the total equity base will be reduced. This reduction will produce a contraction in the equity level of the following period, and therefore a lower demand for investment, labour and loans. If the supply curve of credit does not shift, the interest rate will be reduced, decreasing the burden of financial commitments. The reduction of debt commitments will increase the equity level, thus stimulating demand for labour and investment and restoring the anti-shock equity base level.

Of course, many different types of shock can be taken into account but, as long as they are of transitory type, their effects will be similar to the one I have just analyzed. The adjustment process may be very different (wave-like or linear), depending on the value of $\Gamma^*$. Unlike the Blanchard and Quah (1989) approach, the demand shock will affect the slope of $\Gamma$, thus affecting the dynamic behaviour of the system. In fact, $\delta \Gamma^*/\delta c_p$ is greater than zero: i.e. a positive shock
reduces fluctuations while a negative shock increases them (the asymmetry of shocks helps to explain Blatt’s (1978) finding that time series fluctuations exhibit asymmetric behaviour which cannot be reconciled with the stochastic symmetric approach by Slutsky and Frisch).

I now turn to the so-called supply, or permanent, shock elements. Suppose the coefficient \( w \) is pushed upwards by an innovation: the main effect is a shift of the curve \( \Gamma \) and a steeper slope. As a consequence of the upward shift of \( \Gamma \), due to change in \( \psi \), two different effects have to be taken into account: i) there is an absolute growth of \( a \); ii) fluctuations in \( a \) are reduced (note that there is an asymmetric response to shocks, since the amplitude of fluctuations increases with a negative shock). This case can be well represented by referring to Figure 2. Differently from a temporary shock, a shift of the curve is once and for all (if we ignore the capital accumulation factor, which in any case tends toward zero), but the outcome will be (if \( \Gamma' < 0 \)) that of fluctuating growth. In particular, whenever \( \psi \) increases, total output will grow, therefore driving up the interest rate and equity level. The final effect will be positive, although its size will depend upon the sensitivity of income to the rate of interest and the rate of profit.\(^2\)

3.2 Self-sustained Cycles

We saw above that, when \( \Gamma' < 0 \), a deterministic path may emerge. In this case, if chaotic dynamic conditions hold true, cycles follow an erratic path, i.e. their behaviour is stochastic even if the underlying system is deterministic. In the deterministic setting of our model, a period of increasing activity leads to a higher demand for investment and a higher interest rate. This latter depresses the equity level and reduces activity; therefore, because the equity base drops, the demand for loans and the interest rate will also decrease, thereby stimulating the equity level, production and profits so that the cycle can start again. One of the main criticisms of old-fashioned deterministic cycles (Kaldor, 1940; Hicks, 1950; Goodwin, 1951) was that the cycle was of too regular a period to resemble real-world fluctuations (see Zarnowitz, 1992). The analysis of chaos allows us to overcome this criticism because it reproduces self-sustained cycles of irregular periodicity. The literature contains several tests of chaos (Brock et al., 1991), although the results are far from being definitive, mainly because the economic system evolves structurally, i.e. it can easily shift back and forth between chaotic regimes and fixed points.

Up to now the dynamic resembles the analysis of the endogenous business cycle. I now turn to a different situation and suppose that the propensity to invest is driven by animal spirits:

\[
 b_i = b + b_e
\]

where \( b_e \) is a random disturbance with mean and variance \( (0, \sigma^2) \). Whenever \( b \) changes, the shape of \( \Gamma \) shifts, and \( \Gamma' \) becomes smaller. In particular, fluctuations tend to reduce in persistence and amplitude (because of the positive effect of Pandora’s box: more profits imply more investment and more profits, and so on). Should \( b \) decrease, the system may collapse toward instability, when a cumulative depression sets in, if it is not interrupted by a credit injection or government spending. Of course, these shifts back and forth will produce a succession of types of dynamic behaviour characteristic of each stage, although suddenly qualitative change may also stem from chaotic dynamics (Medio and Gallo, 1992). In particular, whenever \( b \) is close enough to the bifurcation value, each single stochastic shock may produce regime switching, i.e. qualitative change in the dynamic behaviour of the series (Delli Gatti et al., 1993).

4. Growth and Fluctuations

We saw above that the process of fluctuating growth is closely connected to the working of the system. For the sake of analytical convenience, one can divide growth from fluctuations, although the two are closely interdependent. I now show that, whereas growth can be attributed to capital accumulation, technical progress is the cause causes of development.
4.1 Capital Accumulation

In Section 2.3 I showed that, during the adjustment process, capital stock can be accumulated, thus producing the shift of the $\Gamma$ curve. For there to be capital accumulation, the equity base must be greater than zero and net investment must be positive (i.e. $\delta n_{t+1} < \delta a_{t+1}$). If not, capital accumulates. This fact moves the system along a fluctuating growth path because the cyclical component overlaps with capital accumulation. This explains the asymmetric behaviour between the ascending and descending phases of the business cycle when the time series is not detrended: in fact, even if economic fluctuations are symmetric, the trend element introduces a growth factor which biases the series by making the ascending phase last longer than the descending one. Once the trend component has been frozen, the symmetry is restored.

The capital accumulation effect can be identified with shifts in $\varphi_{t}$ in the vertex of $\Gamma$, and in $\Gamma^\prime$. Since $\Gamma^\prime$ does not change if the rate of interest is kept constant, it follows that the dynamic behaviour of the system is not affected by capital accumulation. One may therefore legitimately assert that capital accumulation simply reproduces the dynamic behaviour of the system on a larger scale. This process is, in any case, bounded by floors (no net investment) and ceilings (full employment and credit rationing) which, by changing the initial conditions of the model, will affect its dynamic behaviour (Minsky et al., 1993). If monetary policy is tight, capital accumulation produces an increase in $\Gamma^\prime$, which amplifies fluctuations, since the increase in r augments financial commitments which lowers $\sigma$.

Within a technological paradigm, the growth process is bounded by the full employment barrier. If productivity does not improve, sooner or later the system will hit the full employment barrier, with no possibility of further growth. In order to enjoy a continuous growth process, the system must save on labour inputs. Otherwise the possibility of growth will be limited by the growth of the labour supply or by an increase in participation rates. The empirical evidence suggests that during last century, whereas the capital-output ratio remained almost constant, the capital-labour ratio increased steadily. On the other hand, the participation-rate change had a quite modest role in supporting output growth, and this suggests that what actually increased output was the continuous expansion of the technological frontier.

4.2 Increasing Technological Progress

The analysis set out in this paper supports the Schumpeterian thesis that innovations cause fluctuations. Let us assume that the system is not affected by stochastic disturbances and that no self-sustained cycles arise. In this situation there will be no fluctuations unless a technological improvement is developed. Once the innovation has been introduced, the shape of $\Gamma^\prime$, i.e. $\Gamma^\prime$, will change, producing growth and an adjustment process of fluctuating growth. One can legitimately assert, therefore, that innovations cause fluctuations.

There are several impulses in the real world which affect factor $\psi$: learning by doing (Arrow, 1962), increasing returns (Romer, 1988), and R&D expenditure (Greenwald et al., 1990) are just few of the possibilities that have been investigated. I model change in technological knowledge as:

$$\psi_t = \psi_{t-1} + \gamma (R&D)_{t-1}.$$  

Greenwald et al. (1990), as well as several empirical studies, suggest that R&D activity is linked to equity base and not to bank loans, i.e. $(R&D)_{t-1} < \delta a_{t-1}$. In an asymmetric information setting, external finance is more costly than equity base and it seems likely that risk-averse firms will prefer to finance investment expenditure out of a mix of external finance and equity base, and R&D (whose productivity is relatively random) out of equity base alone. In particular, up to a certain amount of equity base, firms will not spend on R&D, preferring instead to replace depreciated capital stock or to expand production; only if the equity base is large enough will they become involved in a kind of expenditure which is very profitable but whose probability of success is extremely low.

Since $\psi$ enters $\varphi_{t}$ and $\varphi_{t}$, the curve $\Gamma$ will shift and $\Gamma^\prime$ will also change whenever $\psi$ increases. In fact, whenever $\psi$ increases, $\Gamma^\prime$ becomes less steep (changing, perhaps, from a negative to a positive slope), and the vertex also shifts. If $\Gamma^\prime$ changes, the dynamic behaviour involved will also change. I can single out, for analytical reasons, the effect of productive change from short run to long run.

In the short run, fluctuations become less deep and less persistent because the system exploits the change in productivity (as
happened in the first decades after the Second World War, when the business cycle was supposed to have become obsolete because of the high and persistent growth rate) and the growth process enters a virtuous circle. Since the rate of technical change is linked to R&D expenditure, or ultimately to the equity base, a virtuous circle arises. Whenever \( \psi \) increases, the equity base grows. This stimulates the change in technical progress which, in a virtuous loop, stimulates \( a \), and so on. In the long run, this virtuous circle is bound to come to a halt because productivity change will be absorbed by increased wage bills, which depress the equity base and freeze the growth process.

After a certain delay, wages begin to change in order to catch up with productivity increase. This change can be of sufficient magnitude to break the virtuous circle and reduce the rate of growth of the system. If the equity base passes from a period of high \( a \) to one of low \( a \), the increase in productivity slows down, thereby depressing employment, with the consequence that the equity base recovers, and so on, in an alternating process of growth rate. The segmented trends are certainly not due solely to wage variations (population growth or new markets may also affect the trend), but in this context they appear to be the main factor in alternating good and bad periods.

5. Concluding Remarks

According to the most widely-held opinion, when analyzing business fluctuations "most macroeconomists now share the same general analytical approach, that based on the distinction between impulse and propagation mechanisms", namely, the Slutsky-Frisch approach (Blanchard and Fischer, 1989, p. 277) which is basically the equilibrium business-cycle approach. The impulse-propagation approach states that serially uncorrelated shocks, so-called impulses, affect output through a propagation mechanism, i.e. distributed lag relations. Since it has been shown that a simple linear propagation mechanism reproduces the stochastic behavior of the economic time series, and since it is easy to analyze and estimate empirically, the non-linear approach was supplanted by the impulse-propagation mechanism. In the Slutsky-Frisch approach, the system is supposed to be in the steady-state equilibrium position. In order to achieve this steady-state position, one must impose several restrictions: in particular, one must assume that the equilibrium position exists, and is unique and stable, even though it is well known that the last two conditions are still an unresolved puzzle for the general equilibrium theory, and recent work has showed that multiple equilibria may exist if markets are not complete (Cooper and John, 1988).

In this paper, I have described an alternative approach to the analysis of economic growth and fluctuations using a simple stochastic non-linear model which shows a full range of possible outcomes: by integrating stochastic and non-linear dynamics it is possible to offer a new interpretation of the behavior of the time series, where both the Slutsky-Frisch approach and the non-linear tradition can be considered as particular cases. This paper has presented an approach to fluctuations and growth which is corroborated by both empirical evidence and ongoing experience. This framework on a particularly important aspect of real life seems very promising, although further research (Stiglitz, 1993) and econometric investigation is needed to assess its significance.
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