Monetary policy and world commodity markets:  
2000-2007

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1. Introduction

The world economy showed robust economic growth between 2003 and 2007, averaging about 4.5-5.5 percent per year. However, commodity price inflation re-emerged. Commodity prices had their highest rates of increase of the post-war period with the price index of all commodities increasing at a rate of 23 percent per year between 2003 and 2007. Crude oil prices increased fourfold to exceed $90/barrel in October 2007. 1 The US dollar depreciated considerably between 2002 and 2007, with a depreciation of about 63 percent against the euro. Financial markets faced high uncertainty stemming from rising inflationary expectations, credit risk, and a depreciating dollar.

The strong economic growth and accompanying inflationary trends were brought about by expansionary monetary policies in the leading industrial countries, particularly between 2002 and 2004, with central banks forcing interest rates to record lows. Credit expanded at a fast pace in major industrial countries, at the expense of creditworthiness and credit quality, contributing to a rapid increase in aggregate demand for real assets, and for goods and services. While there is no limit to expanding demand for goods and services through credit expansion and unlimited money creation, supply of these goods is, however, constrained in the short run by fixed factors, such as cultivable land or existing plants,

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1 It is assumed here that speculation alone cannot be responsible for persistent trends in commodity prices, and only market fundamentals can support such trends. Speculation can only play a short-term role and is often fuelled by cheap credit.
oil output and other raw materials, and entrepreneurship, and may not follow the expansion of demand, resulting in extreme pressures on prices.\(^2\)

Most striking, consumer price indices (CPIs) in many industrial countries, a leading indicator for the conduct of monetary policy, were not sensitive to large increases in commodity or housing prices. In spite of rapid increases in housing, energy, and food prices, CPIs continued to show small increases, by about 2-3 percent in industrial countries between 2003 and 2007, indicating surprising price stability. This was not the case during the 1970s, when CPIs were highly sensitive to oil shocks and rapid increases in energy prices. The insensitivity of CPIs to commodity prices and to low nominal interest rates may lead policymakers to downplay the risk of inflation.

With monetary policy remaining accommodative and real interest rates being eroded by inflation, commodity price inflationary trends might not subside. An acceleration of inflation rates will certainly slowdown economic growth, and will aggravate financial instability by rapidly eroding the real value of financial assets, and deteriorating the quality of loans. The financial crisis in the sub-prime market in 2007-2008 could be easily traced to lax monetary policy (and lax supervision) with serious financial and economic implications in the real sector.

To bring inflationary trends under control, central banks may have to deliberately reduce the money supply (Friedman, 1959, 1969, and 1972).

\(^2\) Roncaglia (2003) rightly argued that the scarcity of goods is alleviated in the longer run by capital accumulation and technical progress. This point is illustrated by the fact that in 1970 world crude oil output stood at 48 million barrels a day (mbd) and that of natural gas at 1,001.5 billion cubic meters (bcm); in 2007, world crude oil output stood at 87 mbd and natural gas at 3,065.6 bcm, thanks to investment in exploration, development, and refining. Malthusian pessimism regarding land scarcity and diminishing returns certainly did not foresee capital accumulation and technical change, both of which were instrumental for economic growth. Nonetheless, short-run constraints cannot be underestimated. For instance, a bad coffee crop in Brazil will affect coffee supply and prices in the short run. Pressure on oil demand diverted large quantities of grain to ethanol production and fired up food prices in many countries. In the long run, the response would be to invest in capital and research to alleviate both the scarcity of oil and food products. In general, high inflation discourages supply “regardless of the availability of supply”; unclear countries that have experienced hyperinflation have fallen into economic decline and were not able to recover until they overcame the inflationary impediment.
Such a policy would imply a significant increase in interest rates and would necessarily cause a recession and a major debt crisis because, of the size of outstanding loans accumulated during the period of rapid monetary expansion and low creditworthiness, as recently reflected by the sub-prime market; its merit, however, would be to eradicate inflationary dynamics. Monetary authorities will face political conflicts stemming from pressure from debtors to keep inflating the economy in order to increase their wealth and lower their real debt burden, and public pressure to rein in inflation, considered as public enemy number one, and avoid its severe economic and financial dislocation. The evolution of commodity prices, along with other asset prices, such as exchange rates, along with other indicators, should be fully taken into account for sound policymaking and stable growth of the world economy. Neglecting information from commodity prices may result in unsustainable monetary policies.

This paper is organized as follows. In section 2, we review the stance of monetary policy and show that it was excessively expansionary between 2000 and 2007. In section 3, we describe the consequences of this policy on commodity markets. In section 4, we show that CPIs became less responsive to commodity price indices between 2000 and 2007; moreover, their evolution was not in conformity with the Purchasing Power Parity hypothesis. The time series properties of

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4 Kindleberger (2009) described episodes of high asset price inflation and stable or declining wholesale and consumer prices indices in the US between 1927 and 1929, Japan 1985-1989, and Sweden 1985-89; he noted that central banks failed to intervene to arrest asset price inflation, essentially for two reasons: central banks are reluctant to depress economic activity and central banks are concerned about consumer price inflation. Kindleberger pointed out that the consequences of asset price deflation were financial crises and economic turmoil. He implied that monetary policy has to be responsive to asset inflation: “When speculation threatens substantial rises in asset prices, with a possible collapse in asset markets later, and harm to the financial system, or if domestic conditions call for one sort of policy, and international goals another, monetary authorities confront a dilemma calling for judgment, not cookbook rules of the game. Such a conclusion may be uncomfortable. It is, I believe, realistic.” (p. 49).
commodity price indices are studied in section 5, where it is established that these indices were pulled by a common powerful monetary trend. Applying a vector autoregressive (VAR) model, in section 6 we estimate the effects of monetary policy on commodity prices through variance decomposition, and show that interest and exchange rates explain large components of commodity price variance. In section 7, we discuss a forecast of commodity price indices under alternative monetary policy scenarios. We present our conclusions in section 8.

2. Monetary policy from 2000 to 2007

Recent oil shocks and the rapid increase in commodity prices have been fuelled by expansionary policies in the economies of major reserve currency countries between 2001 and 07. More specifically, nominal interest rates fell to record lows for the post-WWII period as depicted in Figure 1. The federal funds rate fell steadily and remained in the range of 1-1.2 percent during the period 2002M12-2004M7, forcing other key interest rates down. The LIBOR six-month dollar rate, fell dramatically and remained within a band of 1.08-1.52 percent during the period 2002M11-2004M5. The three-month euro inter-bank rate fell to 2.03 percent in 2004M3 and was kept within a band of 2.03-2.2 percent during the period 2003M6-2005M10. The three-month US Treasury bill rate fell to a band between 0.90-1.27 percent during the period 2002M11-2004M6. In the same vein, long-term interest rates fell, with the yield on the thirty-year US government bond falling to 4.36 percent in 2003M5. In some key industrial countries, money market rates were near zero between 1999 and 2006.

To force interest rates down, central banks inject liquidity into the economy. Banks attempt to increase their credits and reduce their excess reserves by loaning to higher risk customers in the sub-prime, as demand

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5 For data on interest rates, commodity prices, and exchange rates see the IMF *International Financial Statistics*. 
for credit in the prime market cannot absorb all excess liquidity. In such an operating mode of interest rate targeting, central banks ignore the quality and nature of loans as well as risk factors in order to foster lower interest rates. Most of the excess liquidity is loaned to readily available demand, such as for housing, consumer durables, and short-term credits. It is also loaned to support speculative activities in assets and commodities markets. It rarely finances long-term investment in plants or infrastructure as these types of investment follow a project cycle and are financed through long-term capital from equity or long-term borrowing. Abnormally low interest rates may cause serious misallocation of resources, besides encouraging consumer loans that have no capital backing and face high default risks, reducing marginal efficiency of capital and leading to the selection of low-return investment projects.6


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6 Marginal efficiency of capital, or equivalently, internal rate of return, is introduced by Keynes in *The General Theory of Employment, Interest, and Money*, (1936), page 135. It is defined as the discount rate, which sets present value of prospective returns over the life of an investment equal to the cost of the investment.
Falling interest rates are transmitted to other countries, including major industrial and developing economies, resulting in rapid credit expansion with attendant pressure on demand for real assets, mainly housing, and goods and services. As a result of monetary expansion, the dollar depreciated significantly versus the euro, by about 63 percent, from $0.84 per euro in 2001M6 to $1.37 per euro in 2007M7 (Figure 2); the nominal effective exchange rate of the dollar (NEER) depreciated by about 29 percent during the period 2002M2-2007M7. Such sizeable currency depreciation contributed to increasing pressure on crude oil and other commodity prices, as these prices are quoted in dollars. With inflation trends accelerating in commodity markets, resulting in a fall in real interest rates, monetary policy could be seen as being expansionary.

Historically, a dollar appreciation due to dollar shortage has caused depressed commodities prices.
Moreover, since August 2007 with the decline in the US discount rate and the federal funds rate as well as the injection of liquidity into banks with nonperforming portfolios, monetary policy was further loosened. Such loosening will contribute to further pressure on commodity prices and could in turn bring more instability to financial markets. The combination of low interest rates and double-digit commodity price inflation could seriously weaken financial institutions by eroding the real value of their assets and dissipate the value of international reserves, and may reduce the volume of international trade. It will destroy the value of money, financial savings, and cause a redistribution of wealth in favor of debtors.  

3. Recent trends in commodity prices

Such powerful monetary stimulus resulted in a substantial increase in aggregate demand for goods and services and fueled world real GDP growth, which was reported to have increased at about 4.5-5.5 percent per year between 2003 and 2007. In contrast to previous economic growth cycles, the recent cycle is characterized by rapidly rising commodity prices, with most commodity prices exhibiting double-digit increases during the period 2003M5-2007M7 (Table 1 and Figure 3). The inflationary feature becomes clear when recent oil shocks are compared with earlier shocks. Considering the period 1973M1-1980M12, gold, oil, and natural gas prices increased at a fast pace, 31 percent, 46.5 percent and 29.8 percent per year, respectively; the Commodity Research Bureau (CRB) price index moved at 9.6 percent per year, while food prices increased by 7.2 percent per year. There was, therefore, a distinct energy...
shock that hit specifically oil and natural gas markets during the period 1973M1-1980M12; consequently, the relative price of oil appreciated in relation to other commodities, encouraging energy substitution and conservation.

Table 1 - Commodity price indices, annual percent change, 1973-2007

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<td>Crude oil</td>
<td>46.5</td>
<td>2.0</td>
<td>5.5</td>
<td>30.3</td>
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<tr>
<td>Natural gas</td>
<td>29.8</td>
<td>5.1</td>
<td>41.5</td>
<td>16.7</td>
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<tr>
<td>All commodities</td>
<td>Na</td>
<td>2.5</td>
<td>2.8</td>
<td>24.6</td>
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<tr>
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<td>0.4</td>
<td>19.5</td>
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<td>Gold</td>
<td>31.0</td>
<td>-2.4</td>
<td>4.9</td>
<td>18.5</td>
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<td>Metals</td>
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<td>-2.5</td>
<td>32.9</td>
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<td>0.8</td>
<td>6.2</td>
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<tr>
<td>Food</td>
<td>7.2</td>
<td>-2.4</td>
<td>3.4</td>
<td>9.5</td>
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<tr>
<td>Rice</td>
<td>14.0</td>
<td>-1.5</td>
<td>-4.2</td>
<td>13.1</td>
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<td>Wheat</td>
<td>11.2</td>
<td>-1.7</td>
<td>10.8</td>
<td>14.1</td>
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<tr>
<td>CRB commodity price index</td>
<td>9.6</td>
<td>-0.9</td>
<td>3.0</td>
<td>13.3</td>
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Figure 3 - Commodity price indices, 2000M1-2007M7

Source: IMF, IFS
The periods 1981M1-1999M12 and 2000M1-2003M4 featured commodity price stability. However, with the effect of expansionary monetary policy building momentum and demand expanding, commodity prices were almost uniformly under pressure during the period 2003M5-2007M7, with price increases accelerating to double-digit rates. Paralleling the increase in oil prices, estimated at 30.3 percent per year during the period 2003M5-2007M7, the price index of all commodities rose at 24.6 percent per year during the same period, with non-fuel prices rising at 19.5 percent per year and the price of gold increasing at 18.5 percent per year. Food prices rose rapidly at 9.5 percent, with staple products such as rice, wheat, maize, and cooking oil exhibiting fast price increases. The CRB commodity price index rose at 13.3 percent per year.10

Hence, when oil prices are compared with other prices, they appear to be consistent with the underlying fundamentals for commodity markets, which are characterized by high demand for products, short-term supply constraints, and rapidly increasing commodity prices. Most striking is the simultaneous rise of all commodity prices starting mid-2003, which points to a strong demand shock affecting all commodity markets. If real interest rates are measured against commodity price increases, then they are certainly largely negative and would contribute to

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10 Some widely held views attributed persistently higher commodity prices, including oil, to abnormal commodities demand from developing countries, or major emerging Asian economies (China and India). While distinction of demand by country groupings is irrelevant for world markets, no developing or emerging economy has a reserve currency of its own; therefore, it cannot expand its demand for commodities beyond its international reserves and borrowing capacity. Hence, India cannot buy oil from Saudi Arabia or copper from Chile paying with Indian Rupees. Such trade has to be paid in dollars, euros, or other reserve currencies. Moreover, textbook demand theory makes a clear distinction between relative, nominal prices, and rate of change of prices. For a fixed nominal income and money stock, change in demand will affect relative prices and price levels only. Friedman (1969) showed that for prices and for the exchange rate to sustain a constant or accelerating percent change, the money supply has to increase (or decrease) at a rate exceeding (or below) real GDP growth. Without an accommodative money supply, prices and the exchange rate cannot sustain persistent changes. This dispels the claim that China and India were responsible for constantly higher energy prices, as constant increases (or decreases) in prices can only be a monetary phenomenon. Similarly, depreciating currency can only stem from excessive money supply.
exerting further pressure on commodity prices, and to a significant loss
in real value of loans and savings. Similarly, with the dollar
depreciating against other major currencies, the demand for commodities
will be stimulated significantly. These trends are relevant for oil and
other commodity markets, indicating that commodity markets will be
constantly under pressure unless underlying fundamentals change. In
conjunction with jittery equity markets, and crisis in the housing market,
these indicators show a buildup of inflationary pressures and growing
financial uncertainties. With increasing instability in exchange rates, and
loosening monetary policy following the erosion of real interest rates,
pressure on oil and commodity markets may increase further.

4. Recent trends in consumer price indices

Despite record low interest rates, a sharp depreciation of the dollar,
and simultaneous rise in the prices of most commodities, the CPI measure
of inflation fails to capture these commodity price increases in both the
US and in other industrial countries during the period 2003M5-2007M7.
Instead, CPIs showed remarkable stability and almost no inflationary
pressure, in sharp contrast with experience in the 1970s, when there was a
strong relationship between commodity price increases and CPI inflation
(Table 2 and 3). Besides this weak relationship between CPIs and
commodity prices, CPIs evolution during the period 2003M5-2007M7
deviated persistently from the Purchasing Power Parity hypothesis (Table
4 and Figure 4), in contradiction with the monetary approach to the
balance of payments. The latter predicts that a sharp depreciation of the
exchange rate, holding money supply fixed, will redress external
disequilibrium, and induce an appreciation of currency and a return,
through arbitrage, to long-term purchasing power parity. These two
anomalies in CPIs during the period 2003M5-2007M7 create a price
puzzle whose explanation will help in the design of sound

11 A loan of $80 made in 2002 would buy 4 barrels of crude oil at 2002 prices. If repaid in
October 2007, this loan would buy less than one barrel of crude oil. The proprietor of the
loan would have lost more than ¾ of his real capital.
macroeconomic policies. More specifically, CPIs may induce policymakers to be wrongly reassured about price stability, while commodity prices exhibiting double-digit inflation.

Table 2 - *Consumer price indices, annual percent change, 1973-2007*

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<tr>
<td>CPI US</td>
<td>9.0</td>
<td>3.4</td>
<td>2.6</td>
<td>2.9</td>
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<tr>
<td>CPI industrial countries</td>
<td>10.6</td>
<td>3.7</td>
<td>2.0</td>
<td>2.1</td>
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<tr>
<td>CPI Euro zone</td>
<td>Na</td>
<td>Na</td>
<td>2.2</td>
<td>2.1</td>
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<tr>
<td>CPI World</td>
<td>14.0</td>
<td>15.2</td>
<td>3.9</td>
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<td>24.6</td>
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<tr>
<td>Food prices index</td>
<td>7.2</td>
<td>-2.4</td>
<td>3.4</td>
<td>9.5</td>
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<tr>
<td>CRB Commodity Price index</td>
<td>9.6</td>
<td>-0.9</td>
<td>3.0</td>
<td>13.3</td>
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</table>

*Source*: IMF International Financial Statistics

Table 3 – *Elasticities between commodity and consumer price indices*

Sample period 1973M1-1980M12:

- \( \log(\text{CPI\_US}) = 0.28\log(\text{Oil price index}) + 2.80; \)
  \( t=21.4 \) \( \quad R^2=0.83, \, DW=0.22. \)
- \( \log(\text{CPI\_US}) = 0.94\log(\text{CRB price index}) -1.53; \)
  \( t=14.6 \) \( \quad R^2=0.69, \, DW=0.10. \)
- \( \log(\text{CPI\_Industrial}) = 0.32\log(\text{Oil price index}) + 2.61; \)
  \( t=19.8 \) \( \quad R^2=0.81, \, DW=0.18. \)
- \( \log(\text{CPI\_Industrial}) = 1.08\log(\text{CRB price index}) -2.38; \)
  \( t=13.3 \) \( \quad R^2=0.65, \, DW=0.08. \)

Sample period 2003M5-2007M7:

- \( \log(\text{CPI\_US}) = 0.11\log(\text{Oil price index}) + 4.29; \)
  \( t=20.6 \) \( \quad R^2=0.90, \, DW=0.40. \)
- \( \log(\text{CPI\_US}) = 0.27\log(\text{CRB price index})+3.19; \)
  \( t=13.3 \) \( \quad R^2=0.78, \, DW=0.11. \)
- \( \log(\text{CPI\_Industrial}) = 0.08\log(\text{Oil price index}) + 4.39; \)
  \( t=19.7 \) \( \quad R^2=0.89, \, DW=0.37. \)
- \( \log(\text{CPI\_Industrial}) = 0.20\log(\text{CRB price index}) +3.58; \)
  \( t=14.1 \) \( \quad R^2=0.80, \, DW=0.11. \)
Table 4 - *Unit root test for the real exchange rate of the US dollar per euro, 2000M1-2007M7*

Null Hypothesis: Real Exchange Rate has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=12)

<table>
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<tr>
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<th>t-Statistic</th>
<th>Prob.*</th>
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<tr>
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<td>1% level</td>
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<tr>
<td>5% level</td>
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<tr>
<td>10% level</td>
<td>-2.58393</td>
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*MacKinnon (1996) one-sided p-values

Figura 4 - *Real exchange rate of the US dollar per euro, 2000M1-2007M7*

Source: IMF IFS
4.1. The relationship between commodity and consumer price indices

During the period 1973M12-1980M12, oil prices increased by 46.5 percent per year, causing CPIs in the US and in other industrial countries to rise by 9 percent and 10.6 percent per year, respectively. Worldwide, consumer prices rose by 14 percent per year. However, during the period 2003M5-2007M7, oil prices rose by 30.3 percent per year and all commodity prices rose by 24.6 percent per year, with the CPI unchanged at 2.2 percent per year in the industrial countries, and dropping significantly world-wide to 3.5 percent per year. The relationship between consumer and commodity prices seems to have weakened. In spite of fast increases in oil and non-fuel commodity prices, CPIs remained insensitive, indicating price stability and an absence of any inflationary pressures. Energy and food prices increased dramatically at the retail level in many countries, reflecting money expansion, exchange rates movements, and a rise in commodity prices, yet this fast increase in prices did not translate in a corresponding increase in CPIs.

Our estimation of the relationship between CPIs and commodity price indices during the periods 1973M1-1980M12 and 2003M5-2007M7 (Table 3) shows a sharp drop in the elasticity parameter. The elasticity between the oil price index and the US CPI dropped from 0.28 to 0.11; although highly significant, this elasticity indicated a much smaller effect of crude oil prices on CPIs. The elasticity between the CRB commodity price index and the US CPI fell dramatically from 0.94 to 0.27; although remaining significant, this elasticity indicated a smaller effect of commodity prices on CPIs. The same findings hold with respect to elasticity between the crude oil price index and industrial countries CPIs, falling from 0.32 to 0.08, indicating a smaller effect of high oil prices on CPIs. The elasticity between the CRB commodity price index and industrial countries CPIs also fell from 1.08 to 0.08, showing a smaller effect of commodity price inflation on CPIs in industrial countries.

These regression results may imply a structural change in the relationship between commodity prices and CPIs, with high oil and commodity prices having a much smaller effect on CPIs. There are a
number of possible explanations. First, commodities may account for a much smaller component in the consumer bundle of goods; consequently, an increase in their prices is weighted by a smaller coefficient and has, therefore, a smaller effect on the CPI. Second, as oil and other commodities may be inputs into the production process, productivity gains may reduce the effect of higher commodity prices. Third, productivity gains may also lower the prices of manufactured products, offsetting the impact of higher commodity prices. Fourth, labor costs, particularly in emerging exporting countries with surplus labor, have remained stable. Fifth, given low interest rates, interest costs may have declined, offsetting higher energy and other raw materials costs. Sixth, monetary policy operates through a variable and long lag; it may take about five years for an expansionary monetary policy to have full impact on prices (Friedman, 1969). This reconciliation is only speculative and lacks statistical backing. Further research on the relationship between commodity prices and CPIs seems to be warranted in order to satisfactorily explain this structural change.12

4.2 - Real exchange rate and purchasing power parity

Besides a weakening relationship between CPIs and commodity prices, CPIs evolution during the period 2000M1-2007M7 has been in sharp contrast with long-term purchasing power parity as illustrated by the real exchange rate between the dollar and the euro, defined as the nominal exchange rate of the dollar per euro adjusted by the ratio of the CPI in the Euro zone and in the US, namely:

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12 The use of the CPI as a measure of inflation has long been debated in the literature. Asset prices are considered to be an indicator of future inflation, and ought to be included in a price index measure for more accurate estimate of inflation (See Cecchetti et al., 2000). Blanchard and Clarida (2008) contrasted the macroeconomic effects of oil price shocks for two periods: before 1984 and after 1984. For the period prior to 1984, they showed that oil shocks had a strong impact on consumer price inflation and a depressive effect on output for most industrial countries. However, they claimed that, thanks to structural changes such as competitive labor markets, substantial energy conservation and substitution, high productivity gains, and appropriate money policies, these effects became mild in the post-1984 era.
The real exchange rate during the period 2000M1-2007M7 kept depreciating and showed considerable deviation from the Purchasing Power Parity hypothesis and a weakening of the arbitrage assumption (Figure 4). Unit root tests indicates existence of a unit root in real the exchange rate (Table 4). This finding is in contrast with the prediction of the trade and exchange rate theory. Namely, a substantial depreciation of the exchange rate, holding money supply constant, would lead to reduced imports and increased exports, assuming that the Marshall-Lerner condition holds. The adjustment of the external current account will generate a trade surplus and an appreciation of the currency. Furthermore, higher demand for exports and more expensive imports would increase prices in the depreciating country. Therefore, consequent appreciation of the exchange rate and higher prices in the depreciating country would lead to re-establish the Purchasing Power Parity in the long run. This theoretical prediction seemed to fail. Instead, the exchange rate continued to depreciate without inducing the expected adjustment in prices and the trade balance, which would re-establish the Purchasing Power Parity condition. Such failure may result from an increasing money supply, which keeps the exchange rate under pressure and the trade balance in deficit.

The weakening relationship between commodity prices and CPIs, in conjunction with the persistent deviation of the real exchange rate from the Purchasing Power Parity assumption, points to a puzzle at the level of the CPIs which requires further research, a large decline in interest rates and a fast increase in commodities prices ought to bring about a similar increase in CPIs. Knowing that CPIs are key indicators used by central banks, the inability of these indicators to capture rapid increases in asset and commodity prices and exchange rate movements may mislead policymakers and impose high social and economic costs from the failure to measure inflation and adopt timely policies to control inflationary pressures.
5. A common trend in commodity price indices

The findings reported in Table 1 suggest that commodity prices may be driven by common stochastic trends or common factors during the period 2000M1-2007M7. A model for extracting common trends from reduced form residuals and from structural form residuals is presented in the Appendix. The model draws on integration analysis and decomposes a time series vector of $n$ variables into permanent components and transitory components based on residuals from reduced form or structural forms.

5.1. - Models for extracting common trends

Let $X_t$ be a $(n,1)$ time series vector, composed of $n$ variables, integrated of order one $I(1)$. In order to decompose $X_t$ into a permanent component, representing the common trends part, and a transitory component, an unrestricted vector autoregressive (UVAR) model is assumed for $X_t$, namely:

$$X_t = \mu + A_1 X_{t-1} + \ldots + A_p X_{t-p} + \varepsilon$$

Where: $\varepsilon_t$ is a vector of random shocks assumed to be independently and identically distributed with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t) = \sum$, $\mu$ is a constant, and $A_1, \ldots, A_p$ are $(n,n)$ coefficient matrices with $p$ denoting the lag length. The UVAR can be reformulated in a vector error correction (VEC) form as:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_p \Delta X_{t-p} + \Pi(1) X_{t-1} + \varepsilon_t$$

The common trends representation is a multivariate Beveridge-Nelson decomposition (1981). For univariate models, Beveridge and Nelson (1981) showed that any single integrated ARIMA process has an exactly identified trend plus a transitory component representation, in which the trend is a random walk and the transitory component is covariance stationary.
Π(1) is a \((n,n)\) matrix defined as \(Π(1)=(A_1 + \ldots + A_p-I)\). If the system \(X_t\) is co-integrated with \(k\) unit roots and \(r\) stationary long-run co-integrating relationships, then \(Π(1)\) is a reduced rank matrix and can be decomposed into two matrices \(α_{nxr}\) and \(β_{nxr}\) as follows \(Π(1)=αβ\), where \(β\) denotes a matrix of co-integrating vectors and \(α\) is a matrix of adjustment coefficients. Equation (3) can be written in a vector moving average form (VMA) as a function of \(ε_t\):

\[
X_t = X_0 + C(1)μ_ω t + C(1) \sum_{i=1}^{t} ε_i + C^*(L)e_t = X_0 + C(1)ξ_t + C^*(L)e_t
\]

Where \(ξ_t\) is a \(n\) -dimensional random walk with drift \(μ_ω\), given by \(ξ_t = μ_ω + ξ_{t-1} + ε_t\). The permanent and transitory components of \(X_t\) are, respectively, \(X_t^p = X_0 + C(1)ξ_t\) which is made of a deterministic trend and a stochastic trend, and \(X_t^T = C^*(L)e_t\). Noting that \(C(1)\) has rank \(k<n\) and is written as \(C(1) = β(α_Γβ)^{-1}α_ω\), the common trends driving \(X_t\) are defined by \(k\) combinations of the vector \(ξ_t\), given by \(α_ωξ_t\).

The vector \(X_t\) may be represented as a structural VAR:

\[
B_0X_t = \rho + B_1X_{t-1} + \ldots + B_pX_{t-p} + η_t
\]

Where \(B_0,B_1,\ldots,B_p\) are \((n,n)\) coefficient matrices, \(η_t\) is an \((n,1)\) vector of independent structural shocks with \(E(η_t) = 0\), \(E(η_tη_t^T) = I_n\), and \(η_t = B_0ε_t\). This representation requires identifying restrictions on the elements of \(B_0\). A common trends representation for \(X_t\) would be the following VMA form:

\[
X_t = X_0 + R(1)τ_t + R^*(L)η_t = X_0 + Aτ_t + R^*(L)η_t
\]

Where: \(τ_t\) is a \((k,1)\) vector of common stochastic trends, expressed as a random walk with drift; \(A\) is an \((n,k)\) matrix called the loading matrix, or the long-run multiplier matrix given by \(R(1) = A_{nk},0_{nk}\).
5.2 - Estimating Common Trends in Commodity Price Indices

A vector of four commodity price indices is considered, comprising oil price index, gold price index, non-fuel commodity price index, and CRB price index. Unit root tests show that these price indices where integrated of order one, $I(1)$, during the period 2000M1- 2007M7. Co-integration analyses were performed showing the existence of three co-integrating vectors and one common trend (Table 5).

The VMA model described by equation (4) was applied to estimate a common trend. Cointegration estimation based on the Johansen method yielded the following vector $\alpha' = (0.424135, 0.340266, -0.90279, 1.418373)$. A common trend was computed as $\alpha \xi_t$ and displayed in Figure 5. Hence, a powerful common trend seemed to drive all four price indices, demonstrating that there were no separate shocks hitting specific markets in isolation. The long-run multiplier matrix $A$ in structural VMA (6) was estimated as: $A = (10.47, 6.40, 5.82, 3.07)$. Therefore, in the long run a positive unit permanent shock will increase the oil price by 10.47 units, the gold price by 6.40 units, non-fuel commodities prices by 5.82 units, and the CRB commodities price index by 3.07 units, respectively.

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14 Co-integration analysis for the period 1970M1–1980M12, not reported here, showed the existence of at least two common trends driving commodities prices, which can be qualified as an oil supply shock spreading to commodities markets, and a nominal inflationary shock arising from accommodative monetary policy.

15 Estimated using RATS code written by Henrik Hansen (See Warne, 1993).

16 Co-integration analysis for the period 1970M1-1980M12, not reported here, showed the existence of at least two common trends driving commodities prices, which can be qualified as an oil supply shock spreading to commodities markets, and a nominal inflationary shock arising from accommodative monetary policy.
## Table 5 - Commodity price indices: Johansen Co integration test

<table>
<thead>
<tr>
<th>Hypothesized Number of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace test</th>
<th>Max-Eigenvalue test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistics</td>
<td>Critical Value</td>
<td>Prob. ***</td>
</tr>
<tr>
<td>None *</td>
<td>0.475</td>
<td>87.71</td>
<td>47.86</td>
</tr>
<tr>
<td>At most 1 *</td>
<td>0.24</td>
<td>37.61</td>
<td>29.79</td>
</tr>
<tr>
<td>At most 2 *</td>
<td>0.18</td>
<td>16.08</td>
<td>15.49</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.003</td>
<td>0.28</td>
<td>3.84</td>
</tr>
</tbody>
</table>

*Note:* Trace test indicates 3 cointegrating equations at the 0.05 level; Max-eigenvalue test indicates 3 cointegrating equations at the 0.05 level.

* Denotes rejection of the hypothesis at the 0.05 level.

** MacKinnon-Haug-Michelis (1999) p-values

in structural VMA (6) was estimated as: $A = (10.47, 6.40, 5.82, 3.07)$. Therefore, in the long run a positive unit permanent shock will increase the oil price by 10.47 units, gold price by 6.40 units, non-fuel commodities prices by 5.82 units, and the CRB commodities price index by 3.07 units, respectively.

The common trend driving commodity price indices during the period 2000M1-2007M7 can be attributed to the lag effect of expansionary monetary policy and can be characterized as a demand shock. With real interest declining, when measured in terms of commodity prices, and with an expansion of credit, real aggregate demand for goods and services has been constantly pushed upward, creating tensions in commodities markets and pushing prices constantly upward. As mentioned above, real economic growth was boosted to about 4.5-5.5 percent per year between 2003 and 2007, creating higher demand for commodities. In view of short-term supply constraints, most of the market clearing was born by prices. Higher prices act to reduce real cash balances and to depress demand temporarily. Higher prices may,
but not necessarily, act to increase the short-term supply of commodities. However, as monetary policy remained expansionary or accommodative, more credit expansion and higher money supply supported further demand expansion, which seemed to dominate the price and supply effects and to constantly push prices upward.

6. Role of monetary policy in commodities markets

In this section, the role of monetary policy in commodity markets is examined using a VAR approach. Four VARs were considered for studying the impact of interest and exchange rates on commodity prices.
VAR 1 comprised the oil price index, the LIBOR, and the NEER; VAR 2 comprised the gold price index, the LIBOR, and the NEER; VAR 3 comprised the non-fuel commodity price index, the LIBOR, and the NEER; and VAR 4 comprised the CRB price index, the LIBOR, and the NEER. In each VAR, the transmission channel from the LIBOR and the NEER to commodity prices was through changes in demand and supply for the respective commodity induced by changes in the LIBOR and the NEER; the market clearing commodity price would depend on the extent of excess demand and demand and supply price elasticities characterizing each commodity market. In each VAR, the effect of the LIBOR and the NEER on each commodity price was analyzed in terms of variance decomposition (Figure 6).

In VAR 1, the optimal lag using the Akaike information criterion was found to be 20 months. Variance decomposition shows that the effect of the interest rate builds up quickly and could explain up to 20 percent of the oil price variance at a horizon of 3 months and about 41 percent at a horizon of 30 months. Similarly, the NEER effect builds up quickly and could explain up to 25 percent of the oil price variance at a horizon of 7 months, and remains an important component at later horizons, explaining about 10 to 20 percent of the oil price variance. In VAR 2, the optimal lag was chosen at 20 months. Variance decomposition shows a predominant role for interest and exchange rates in gold price movements. The impact of the LIBOR of the price of gold builds up very quickly and could explain up to 35 percent of the gold price variation in gold prices within a horizon of 2 months and up to 50 percent at a horizon of 8 months. The LIBOR remains a determinant variable at later horizons explaining between 60 to 70 percent of the gold price variance. The exchange rate turns out to be a dominant factor in the dynamics of the price of gold change, with its effect building up rapidly to explain about 58 percent of the gold price variance at a horizon of 6 months. The NEER remains an important component of the change in the price of gold at later horizons accounting for 22 to 37 percent of this variance.
In VAR 3, the optimal lag was chosen at 20 months. Variance decomposition shows that the LIBOR explains up to 70 percent of the variation of the non-fuel commodities price index and remains a main component in this price variation. The NEER, however, explains a small portion of the non-fuel commodities price changes. In VAR 4, the optimal lag was chosen at 20 months. Variance decomposition shows that the LIBOR can explain about 41 percent of the CRB price index variance at a horizon of 11 months, and remains an important component at later horizons, explaining about 22 to 27 percent of this variance. The NEER plays a small role, about 8 to 10 percent in the explanation of the variance of the CRB price index.

Variance decomposition shows that monetary policy was important in commodity price movements, explains a large portion of these

Figure 6 - Commodity price indices, variance decomposition
movements. The LIBOR accounted for a large part of the variance of the four commodity price indices. The exchange rate had an influential role in gold price variance; its role remained important in oil price changes, however, its effect became small in the case of non-fuel commodity and CRB price indices.

7. Forecasting commodity price trends

Based on recently observed commodity prices, we examine hypothetical developments in commodity price indices under alternative monetary policy scenarios: a baseline scenario where present monetary stance is maintained and an alternative scenario, which assumes monetary tightening.

If loose monetary policy continues its trend, key interest rates will continue to fall, in real terms, thus boosting aggregate demand for commodities further. The sale of assets, such as housing, will depend, not on savings, but on loans. Money demand will be reduced significantly to avoid inflationary cost. Under this scenario, inflationary pressures will increase as illustrated by Figure 7, which is obtained from a forecast based on the persistence of common trend analyzed in Section 5. Exchange rate instability caused by an expansionary monetary policy will erode real value of international reserves, and may weaken international trade. The world economy may enter an inflationary-recessionary cycle, with real output growth decelerating and commodity prices continuing to spiral upward. This outcome can be illustrated by the cut of 50 basis points in the federal funds rate in September 2007; consequently, by early October 2007, oil prices jumped by 20 percent to cross $90/barrel in October 2007, gold prices rose by 12 percent to $762 per ounce, and the US dollar fell to $1.43/euro. Further cuts in interest rates between August 2007 and December 2008 and liquidity injections pushed oil prices to $147/barrel in July 2008 and food prices to riot levels in many vulnerable countries.

An alternative scenario would be tightening monetary policy to rein in inflation. This scenario assumes Kindleberger’s view that central banks
ought to react to asset price inflation even though wholesale or consumer price indices might be stable or even declining. If this scenario materializes, the world economy would witness a cooling off in commodity prices. Such

Figure 7 - *Forecasting commodities prices under loose monetary policy*

![Chart showing commodity price forecasts](image)

*Note:* Pbind=crude oil price index, Goldind=gold price index, nfind =nonfuel commodities price index, Crb=Commodity Research Bureau price index.

scenario would require major central banks to change operating procedures by strictly controlling monetary aggregates instead of controlling money market interest rates. If central banks decide to rein in money supply in order to choke off inflation, as was the case between 1979 and 1982, then nominal interest rates will jump to high levels. As real aggregate demand decelerates under the influence of higher real interest rates, a recessionary cycle will take place. In turn, demand for commodities will be checked. Under this scenario, inflationary pressure may be subdued.
This scenario is best illustrated by looking at Figure 8, particularly at the period 1979-1982, when major central banks decided to control monetary aggregates instead of controlling interest rates. Such a decisive tightening of monetary policy brought interest rates to high levels, with the federal funds rate and the LIBOR reaching 19 percent and 18 percent in 1981M7, respectively. Dollar exchange rates appreciated considerably, with the US NEER reaching a historical peak of 138 in 1985M3. Following this strong tightening, inflation rates came down quickly to an average of 3.5-4 percent a year between 1981 and 1999 in both the US and industrial countries.

Figure 8 - Federal funds rate, Libor, and Neer, 1970-2007

Source: IMF IFS

Implications of a tight monetary scenario for commodities can be examined by considering actual data for 1980M1-1999M12 (Figure 9). Under such a scenario, oil prices were forced to trend persistently downwards, losing about 50 to 60 percent of their appreciation between 1982 and 1985. Similarly, gold prices were most sensitive to monetary policy tightening and falling steeply by about 50 percent between 1981 and 1982. Non-fuel commodity prices trended
downward persistently, falling by about 30 percent between 1981 and 1983. The CRB price index also trended down progressively, falling by about 20 percent between 1981 and 1983. The pace and extent of the commodity price decline depends on the degree of monetary policy tightness.

Figure 9 - Actual commodity price indices under tight monetary policy, 1980-1999

Figure 9 illustrates that commodity prices could be stabilized by a tight monetary policy, with demand brought in line with supply of these commodities. Figure 9 indicates only expected trends under a tight monetary policy scenario and cannot be applied systematically to forecast commodity prices in 2007 as the nature of shocks were different and the number of common trends was also different from those in the 1970s. More importantly, the degree of erosion in real interest rates could be substantial between 2003 and 2007 compared with real interest rates in the late 1970s when commodity price inflation was less severe than the one witnessed between 2003 and 2007. The degree of adjustment in
prices will depend on the ability of monetary policy to turn real interest rates positive.

A scenario of tight monetary policy could be simulated with equation (6) by assigning negative values to permanent shocks over a forecast period. Such a scenario indicates only the direction of adjustment. The degree of monetary tightness will be determined gradually until commodity prices start responding persistently to new economic fundamentals, as illustrated by the 1980-1982 monetary policy episode when the federal funds rate and the LIBOR kept increasing until downward persistence in commodity prices became noticeable and price stability was achieved. Based on the long-run multiplier matrix, $A = (10.47, 6.40, 5.82, 3.07)$, a negative impulse of one unit would bring down oil prices over the long-run by 10.47 units, gold prices by 6.40 units, non-fuel commodities prices by 5.82 units, and the CRB commodities price index by 3.07 units, respectively. A bigger negative shock would bring down commodity prices by a multiple of the long-run coefficients.

The conduct of a tight monetary policy will be opposed by debtors and investors as the economy has become heavily dependent on borrowing and money creation. Any small credit squeeze, under these conditions of heavy dependence on credit expansion and inflation, will stifle speculative activities, increase the debt burden, and sharply reduce demand for assets, such as housing and durable goods. The conduct of a tight monetary policy would require central banks to be immune from pressures and aim at safeguarding the safety and stability of the financial system and the value of money as a medium of exchange and store of value.

8. Conclusion

Recent trends in commodity prices have been problematic. By sustaining an increase at 24.6 percent per year during the period 2003M5-2007M7, commodity prices became highly inflationary and caused prices to increase rapidly in most countries. We have shown that the
simultaneous increase in all prices during the period 2003M5-2007M7 can only be a monetary phenomenon and was the delayed effect of an overly expansionary monetary policy, which led to a fast expansion of all types of credit, irrespective of creditworthiness, and to a strong expansion of demand for real assets, goods, and services. In view of short-term supply constraints, commodity prices moved rapidly in response to large excess demand. In particular, there was no specific shock confined to a single commodity market, such as an oil shock; instead, all commodity markets were under the same shock, identified as a monetary shock.

Monetary stance has been loose, mainly as real interest rates were eroded by inflation, and inflationary expectations have become self-fulfilling. Maintaining the present monetary stance would cause further rises in commodity prices, and could result in a severe world recession and disorderly financial markets. In order to rein in inflation and bring back a measure of stability in commodity and financial markets, monetary policy has to be tightened considerably and be directed tostrictly controlling money supply (Friedman, 1959). A tightened monetary policy would necessarily cause a tremendous increase in interest rates, a debt crisis given the low quality and high volume of loans, and a temporary recession as illustrated by the 1979-1982 episode; however, its merit would be to uproot inflation and stabilize markets. In

18 Views regarding the recent housing and commodity price bubbles have been controversial, with central bankers and a large recent literature refuting any link between expansionary money policy and asset price bubbles. However, Frankel (2008) sharply criticized the views of central bankers that dismissed any influence for monetary policy on commodity price inflation. He maintained that commodity price inflation between 2002 and 2008 was attributable to low interest rates. He pointed out that interest rates are the major determinant of commodity prices. His regression analysis established a highly significant negative effect of interest rates on the composite index of all commodity prices, as well as on individual commodity price indices. He argued that low interest rates increase the demand for storable commodities and decrease their supply. The impact comes through a variety of channels: (i) by reducing the incentive for extraction today rather than tomorrow; (ii) by increasing the incentive for firms to carry inventories; and (iii) by encouraging speculators to shift from low-yield assets, such as bonds, and into high-yield commodity contracts. Speculators borrow at low short-term interest rates to purchase a long-term asset at a price above its long-run expected value. Speculators know that the asset price will fall when the central bank raises the interest rate at some future date, but in the meantime they can earn income above their cost of borrowing; the excess income is known as the “carry.”
sum, the world economy faces a dilemma: maintaining the present course of monetary policy would ruin the real value of financial assets and international reserves, may discourage commodities supply, and would turn out to be recessionary. If the course of monetary policy is to be corrected, through controlling the money supply, interest rates will go up sharply, exchange rates will appreciate, another debt crisis may erupt, and a temporary recession may set in as was the experience in 1979-1982. The merit of a prudent monetary policy would be to bring back price stability and durable economic growth, as illustrated by episodes between 1983 and 1999.

APPENDIX

Let $X_t$ be a $(n,1)$ time series vector, composed of $n$ variables, integrated of order one $I(1)$. In order to decompose $X_t$ into a permanent component, representing the common trends part, and a transitory component, an unrestricted vector autoregressive (UVAR) model is assumed for VAR for $X_t$ namely:

$$X_t = \mu + A_1X_{t-1} + ... + A_pX_{t-p} + \varepsilon_t$$  \hspace{1cm} (A.1)

where $\varepsilon_t$ is a vector of random shocks assumed to be independently and identically distributed with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t, \varepsilon_t) = \Sigma$, $\mu$ is a constant, and $A_1, ..., A_p$ are $(n,n)$ coefficient matrices with $\rho$ denoting the lag length. The UVAR can be reformulated in a vector error correction (VEC) form as:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + ... + \Gamma_p \Delta X_{t-p} + \Pi(1) X_{t-1} + \varepsilon_t$$  \hspace{1cm} (A.2)

where: $\Gamma_p = -A_p, \Gamma_{p-1} = -(A_p + A_{p-1}), ..., \Gamma_1 = -(A_p + A_{p-1} + ... + A_1)$, $\Pi(1)$, is a $(n,n)$ matrix defined as $\Pi(1) = (A_1 + ... + A_p - I)$. If the system $X_t$ is co-integrated with $k$ unit roots and $r$ stationary long-run co-integrating relationships, then $\Pi(1)$ is a reduced rank matrix and can be decomposed into two matrices $\alpha_{nxr}$ and $\beta_{nxr}$ as follows $\Pi(1) = a\beta'$, where $\beta$ denotes a
matrix of co-integrating vectors and $\alpha$ is a matrix of adjustment coefficients. By Engle-Granger representation theorem (1987), the VEC admits a reduced form Wold vector moving-average representation (VMA) in terms of the shocks $\varepsilon_t$:

$$\Delta X_t = \mu_w + C(L)\varepsilon_t$$  \hspace{1cm} (A.3)

Where: $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is a $(n,n)$ lag polynomial matrix, $C_0 = I_n$, $C(1)$ is a reduced-rank $(n,n)$ matrix with rank $k < n$, satisfying $\mu_w = C(1)\mu$, $\beta C(1) = 0$, and $C(1)\alpha = 0$. Let $\alpha_\perp$ and $\beta_\perp$ with dimensions $(n,k)$ denote the orthogonal complements of $\alpha$ and $\beta$ defined as $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$ \(^{19}\), then $C(1)$ can be written as $C(1) = \beta_\perp(\alpha_\perp\Gamma\beta_\perp)^{-1}\alpha_\perp$, where $\Gamma = \Gamma_1 + \ldots + \Gamma_{p-I}-I$. Let $C(L) = C(1)+(1-L)C^*(L)$, $C^*_t = -\sum_{j=i+1}^{\infty} C_j$, the solution to the difference equation (4) can be written in levels as a function of $\varepsilon_t$:

$$X_t = X_0 + C(1)\mu_w t + C(1)\sum_{i=1}^{T} \varepsilon_i + C^*(L)\varepsilon_t = X_0 + C(1)\xi_t + C^*(L)\varepsilon_t \hspace{1cm} (A.4)$$

Where: $\xi_t$ is a $n$-dimensional random walk with drift $\mu_w$, given by $\xi_t = \mu_w + \xi_{t-1} + \varepsilon_t$. The permanent and transitory components of $X_t$ are, respectively, $X^P_t = X_0 + C(1)\xi_t$, which is made of a deterministic trend and a stochastic trend, and $X^T_t = C^*(L)\varepsilon_t$. Noting that $C(1)$ has rank $k < n$ and is written as $C(1) = \beta_\perp(\alpha_\perp\Gamma\beta_\perp)^{-1}\alpha_\perp$, the common trends driving $X_t$ are defined by $k$ combinations of the vector $\xi_t$, given by $\alpha_\perp^T\xi_t$.

Note that $\varepsilon_t$ are reduced form errors and are combinations of structural shocks denoted by $\eta_t$, which can be identified with the help of a structural VAR, expressed as

$$B_0 X_t = \rho + B_1 X_{t-1} + \ldots + B_p X_{t-p} + \eta_t \hspace{1cm} (A.5)$$

\(^{19}\) Consider the orthogonal projection of $\alpha$ denoted as $\alpha_\perp^p = I_{n\times n} - \alpha(\alpha'\alpha)^{-1}\alpha'$ satisfying $\alpha'\alpha_\perp^p = 0$, then $\alpha_\perp$ with dimension $(n,k)$ can be obtained as any linear combination of the columns of $\alpha_\perp^p$. Similarly for $\beta_\perp$. 

Where: $B_0, B_1, ..., B_p$ are $(n,n)$ coefficient matrices, $\eta_t$ is an $(n,1)$ vector of independent structural shocks with $E(\eta_t) = 0$, $E(\eta_t \eta_t') = I_n$ and $\eta_t = B_0 \varepsilon_t$. The MVA representation for structural VAR is given by:

$$\Delta X_t = \rho_w + RL \eta_t$$ \hspace{1cm} (A.6)

In view of the cointegration relations, the structural shocks $\eta_t$ are partitioned into $\varphi_t$, a $(k,1)$ vector of permanent shocks, and $\psi_t$, a $(r,1)$ vector of temporary shocks, i.e. $\eta_t = (\varphi_t \psi_t)$. After expressing the MVA in terms of structural shocks, King et al. (1991) proposed the following common trends representation:

$$X_t = X_0 + R(1) \tau_t + RL \eta_t = X_0 + A \tau_t + RL \eta_t$$ \hspace{1cm} (A.7)

Where: $\tau_t$ is a $(k,1)$ vector of common stochastic trends, expressed as a random walk with drift given by $\tau_t = a + \tau_{t-1} + \varphi_t$; $A$ is an $(n,k)$ matrix called the loading matrix, or the long-run multiplier matrix given by

$$R(1) = (A_{nxk} 0_{nxk}), \text{ and } R^*(L) = C^*(L)B^{-1}_{0}.20$$

Hence, common trends are defined as $k$ combinations of either reduced form shocks $\varepsilon_t$ or structural permanent shocks $\varphi_t$, where the combination matrix are $\alpha'_\perp$ or $A$, respectively. Both $\varepsilon_t$ and $\eta_t$ are unobserved variables. Consequently, Gonzalo and Granger (1995) preferred to construct common trends using observed statistical data $X_t$, instead of estimated shocks $\hat{\varepsilon}_t$ or $\hat{\eta}_t$. Noting the identity $\beta'_\perp (\alpha'_\perp \beta)_{-1}^{-1} \alpha'_\perp + \alpha(\beta'\alpha)^{-1} \beta' = 1$, they expressed $X_t$ as:

$$X_t = \omega_1 \alpha'_\perp X_t + \omega_2 \beta'X_t$$ \hspace{1cm} (A.8)

---

20 King et al. (1991)suggested that the matrix $A$ be written as: $A = A_0 \pi$, where $A_0$ is a $(n,k)$ matrix with parameters chosen so that $\beta' A_\beta = 0$, and where the free parameters of $A$ are lumped into the $(k,k)$ matrix $\pi$ given by: $\pi \pi' = (A'_0 A_0)^{-1} A'_0 C(1) \Sigma C(1)' A_0 (A'_0 A_0)^{-1}$. The matrix $\pi$ can be determined from a Choleski decomposition of $\pi \pi'$. Given the estimate $\hat{\pi}$, $A$ is fully identified as: $\hat{A} = A_0 \hat{\pi}$. 
Where: $\omega_1 = \beta_\alpha (\alpha' \beta_\alpha)^{-1}$ and $\omega_2 = \alpha (\beta' \alpha)^{-1}$. Their common trends are $k$ linear combinations of $X_t$, given by $\alpha' X_t$; whereas, their transitory components are given by the cointegrating relations $\beta' X_t$. By ignoring shocks, Gonzalo and Granger’s (1995) approach does not permit the simulation of the impact of policy shocks on $X_t$, a major drawback, which explains its limited use in VAR literature.

REFERENCES


